3. Polarization

-Dr. A. S. Padampalle

The phenomena of reflection, refraction, interference, diffraction are common to both transverse waves and longitudinal waves. But the transverse nature of light waves is demonstrated only by the phenomenon of polarization.

Polarization is the restriction of vibrations of light wave to a single plane.

1.1Polarisation of transverse waves: Let a rope AB be passed through two parallel vertical slits S_1 and S_2 placed close to each other. The rope is fixed at the end B. If the free end A of the rope is moved up and down perpendicular to its length, transverse waves are generated with vibrations parallel to the slit. These waves pass through both S_1 and S_2 without any change in their amplitude. But if S_2 is made horizontal, the two slits are perpendicular to each other. Now, no vibrations will pass through S_2 and amplitude of vibrations will become zero. i.e the portion S_2B is without wave motion as shown in fig 1.

On the other hand, if longitudinal waves are generated in the rope by moving the rope along forward and backward, the vibrations will pass through S_1 and S_2 irrespective of their positions. This implies that the orientation of the slits has no effect on the propagation of the longitudinal waves, but the propagation of the transverse waves, is affected if the slits are not parallel to each other. A similar phenomenon has been observed in light, when light passes through a tourmaline crystal.

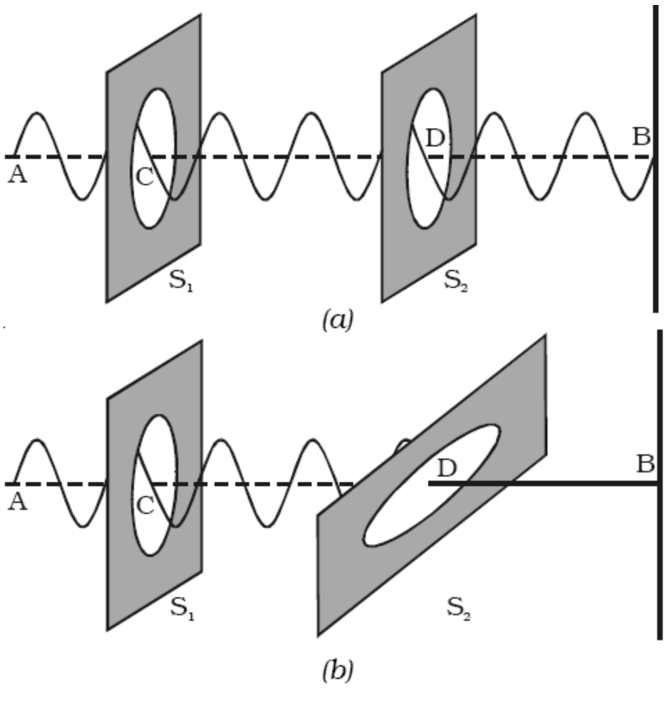


Figure 1 Polarisation of transverse waves

Light from the source is allowed to fall on a tourmaline crystal which is cut parallel to its optic axis (Fig. 2). The emergent light will be slightly coloured due to natural colour of the crystal. When the crystal A is rotated, there is no change in the intensity of the emergent light. Place another crystal B parallel to A in the path of the light. When both the crystals are rotated together, so that their axes are parallel, the intensity of light coming out of B does not change. When the crystal B alone is rotated, the intensity of the emergent light from B gradually decreases. When the axis of B is at right angles to the axis of A, no light emerges

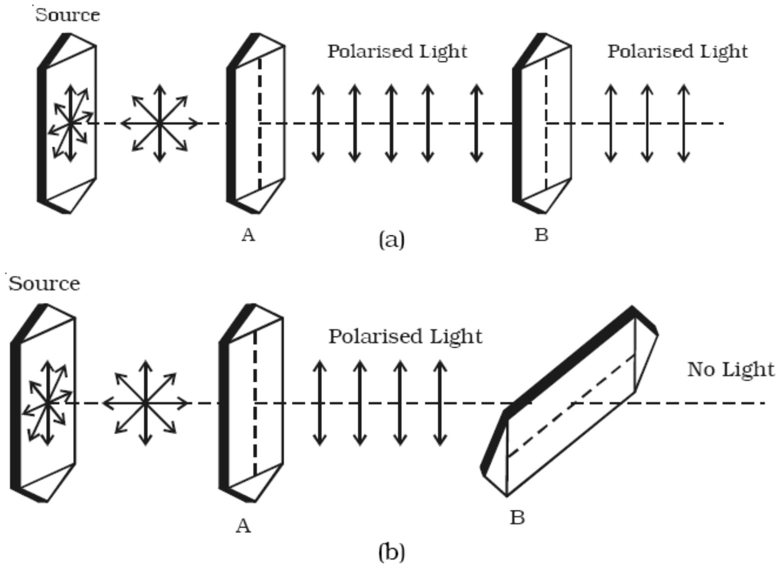


Figure 2 Polarisation of transverse waves

from B (Fig. 2.b). If the crystal B is further rotated, the intensity of the light coming out of B gradually increases and is maximum again when their axis are parallel.

Comparing these observations, it is concluded that the light waves are transverse in nature. Light waves coming out of tourmaline crystal A have their vibrations in only one direction, perpendicular to the direction of propagation. These waves are said to be polarised. Since the vibrations are restricted to only one plane parallel to the axis of the crystal, the light is said to be plane polarised. The phenomenon of restricting the vibrations into a particular plane is known as polarisation.

Ordinary light from a source has very large number of wavelengths. The orientations of electric vectors are random and are not confined to a plane. Hence ordinary light is unpolarised.

1.2 Polariser and Analyser

A device which produces plane polarised light is called a polariser. A device which is used to examine, whether the light is plane polarised or not is called an analyser. A polariser can serve as an analyser and vice versa.

A ray of light is allowed to pass through an analyser. If the intensity of the emergent light does not vary, when the analyser is rotated, then the incident light is unpolarised; If the intensity of light varies between maximum and zero, when the analyser is rotated through 90^{o}, then the incident light is plane polarised; If the intensity of light varies between maximum and minimum (not zero), when the analyser is rotated through 90°, then the incident light is partially plane polarised.

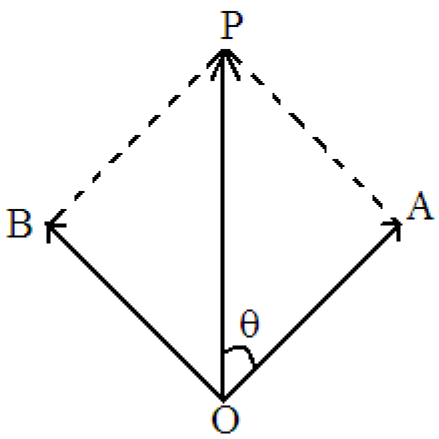
1.3 Malus Law:

Malus law states that the intensity of the polarized light transmitted through the analyzer varies as the square of the cosine of the angle between the plane of transmission of the analyzer and the plane of the polarizer.

Proof: The polarized vibration may be resolved into two rectangular components:

i) parallel to the plane of transmission of the analyzer,

ii) At right angles to it.



Let OP = a be the amplitude of the vibrations transmitted or reflected by the polarizer

θ is the angle between the planes of the polarizer and the analyzer.

Resolve OP into two components,

i) $a\ cos\theta$ along OA and

ii) $a\ sin\theta$ along OB.

Only $a\ cos\theta$ components is transmitted through the analyzer.

We know that, intensity of light is directly proportional to square of amplitude.

$$\therefore E_1 = (a\ cos\theta)^2 = a^2 cos^2\theta$$

But $E = a^2$ is the intensity of incident polarized light.

$$E_1 = Ecos^2\theta \text{ and } E_1 \propto cos^2\theta$$

This is proof of Malus law.

Case I: When θ=0 i.e. the two planes are parallel then

$$E_1 = E\ cos0 = E$$

Case II: When θ = π/2, the two planes are at right angle to each other.

$$\therefore E_1 = E\ (\ cos\ {}^{\pi}/_{2})^2 = 0$$

1.4 Double refraction:

When a ray of light is refracted by a crystal like calcite it gives two refracted rays. This phenomenon is called double refraction. Hence, two images of a single object are formed. This phenomenon is exhibited by several other crystals like quartz, mica etc.

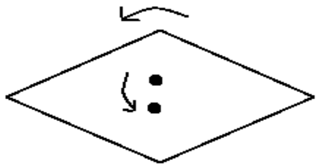


Fig. (a)

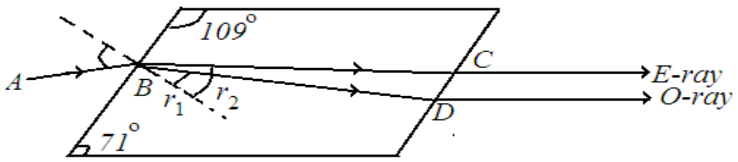
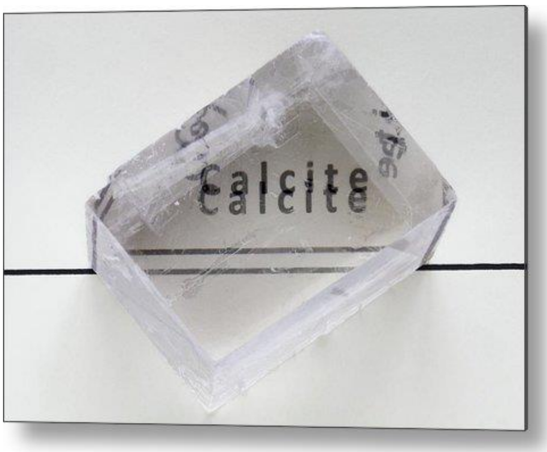


Fig. (b)

The phenomenon of double refraction can be explained with help of following experiment.



Mark an ink dot on a piece of paper. Place a calcite crystal over this dot on the paper. Two images will be observed. Now rotate the crystal slowly as shown in Fig. (a). Place your eye vertically above the crystal. It is found that one image remains stationary and the second image rotates with the rotation of the crystal. The stationary image is known as the ordinary image while the second image is known as the extraordinary image.

When a ray of light AB is incident on the calcite crystal making an angle of incidence = *i*, it is refracted along two paths inside the crystal,

i) along BC making an angle of refraction $=r_2$ and

ii) along BD making an angle of refraction= r_1.

These two rays emerge out along DO and CE which are parallel as shown in Fig. (b)

The ordinary ray has a refractive index $\mu_0 = \frac{sini}{sinr_1}$ and the extraordinary ray has a refractive index$\mu_e = \frac{sini}{sinr_2}$. It is found that the ordinary ray obeys the laws of refraction and its refractive index is constant. In the case of the extraordinary ray, its refractive index varies with the angle of incidence and it is not fixed.

In the case of calcite crystal $\mu_o > \mu_e$, because r_1 is less than r_2. Therefore the velocity of light for the ordinary ray inside the crystal will be less compared to the velocity of light for the extraordinary ray. In calcite, the extraordinary ray travels faster as compared to the ordinary ray. Moreover the velocity of the extraordinary ray is different in different directions because its refractive index varies with the angle of incidence.

It has been found that both the rays are plane polarized. The vibrations of the ordinary ray are perpendicular to the principal section of the crystal while the vibrations of the extraordinary ray are in the plane of the principal section of the crystal. Thus, the two rays are plane polarized, their vibrations being at right angles to each other.

1.5 Nicol Prism

It is an optical device used for producing and analyzing plane polarized light.

It was invented by William Nicol, in 1828. We know that, when a beam of light

is transmitted is transmitted through a calcite crystal, it breaks up into two rays :

(1) the ordinary ray which has its vibrations perpendicular to the principal section of the crystal and (2) the extraordinary ray which has its vibrations parallel to the principal section.

The nicol prism is made in such a way that it eliminates one of the two rays by total internal reflection. It is generally found that, the ordinary ray is eliminated and, only the extraordinary ray is transmitted through the prism.

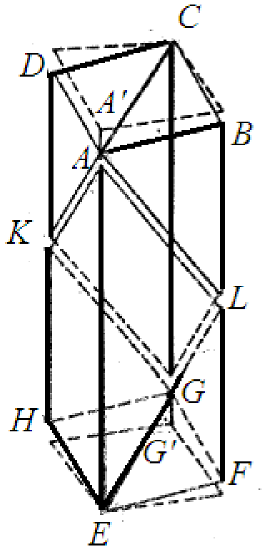


Fig. (a)

A calcite crystal whose length is three times its breadth is taken. Let A'BCDEFG′H represent such a crystal having A′ and G' as its blunt corners and A′CG′E is one of the principal sections with ∠A′CG′ = 70°. The faces A'BCD and EFG'H are ground in such a way that the angle ACG becomes = 68^0 instead of 71^0. The crystal is cut along the plane AKGL, as shown in Fig. (a). The two cut surfaces are grounded and polished optically flat and then cemented together by Canada balsam whose refractive index lies between the refractive indices for the ordinary and extraordinary rays for calcite.

Refractive index for the ordinary $\mu_O = 1.658$

Refractive index for the Canada balsam $\mu_B = 1.55$

Refractive index for the extraordinary $\mu_{E = 1.486}$

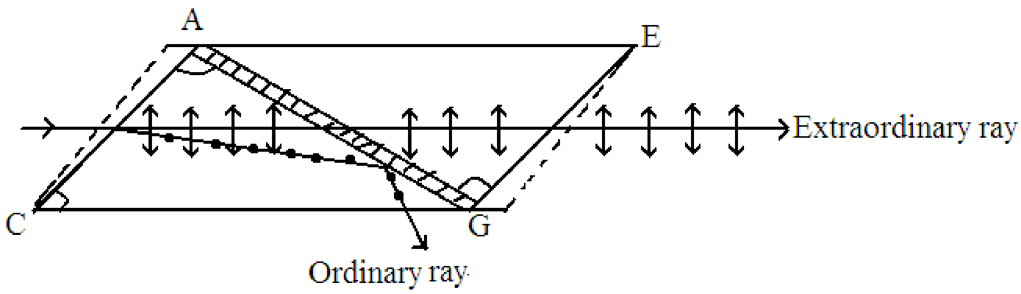


Fig. (b)

In Fig. (b), the section ACGE of the crystal is shown. The diagonal AG represents the Canada balsam layer. It is clear that Canada balsam acts as a rarer medium for an ordinary ray and it acts as a denser medium for extraordinary ray. Therefore, when the ordinary ray passes from a portion of the crystal into the layer of Canada balsam it passes from a denser to a rarer medium. When the angle of incidence is greater than the critical angle, the ray is totally internally reflected and is not transmitted. The extraordinary ray is not affected and is transmitted through the prism.

The working of the prism is explained as follows;

i) Refractive index for ordinary ray with respect to Canada balsam is

$$\mu = \frac{1.658}{1.550}$$

$$\therefore sin\theta = \frac{1}{\mu} = \frac{1.550}{1.658}$$

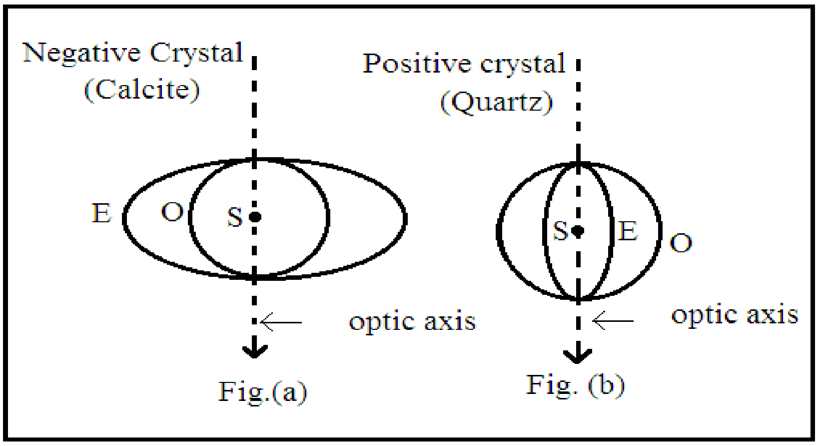
$$\therefore \theta = 69^{\circ}$$

If the angle of incidence for the ordinary ray is more than the critical angle, it is totally internally reflected and only the extraordinary ray passes through the nicol prism. Therefore, a ray of unpolarized light on passing through the nicol prism in this position becomes plane-polarized.

ii) If the angle of incidence is less than the critical angle for the ordinary ray, it is not reflected and is transmitted through the prism. In this position both the ordinary and the extraordinary rays are transmitted through the prism.

1.6 Huygens Explanation of double refraction in uniaxial crystals

Huygens explained the phenomenon of double refraction with the help of his principle of secondary wavelets. A point source of light in a double refracting medium is the origin of two wavefronts. For the ordinary ray, for which the velocity of light is the same in all direction the wavefront is spherical. For the extraordinary ray, the velocity varies with the direction and the wavefront is an ellipsoid of revolution. The velocities of the ordinary and the extraordinary rays are the same along the optic axis.



Consider a point source of light *S* in a calcite crystal as shown in Fig.(a). The sphere is the wave surface for the ordinary ray and the ellipsoid is the wave surface for the extraordinary ray. The ordinary wave surface lies within the extraordinary wave surface, such crystals are known as negative crystals. For crystals like quartz, the extraordinary wave surface lies within the ordinary wave surface which is known as positive crystals. Positive crystals is as shown in Fig. (b).

i) For the negative uniaxial crystals, $\mu_o > \mu_E$. The velocity of the extraordinary ray varies as the radius vector of the ellipsoid. It is least and equal to the velocity of the ordinary ray along the optic axis but it is maximum at right angles to the direction of the optic axis.

ii) For the positive uniaxial crystals, $\mu_E > \mu_o$. The velocity of the extraordinary ray is least in a direction at right angles to the optic axis. It is maximum and is equal to the velocity of the ordinary ray along the optic axis. Hence, from Huygens' theory, the wavefronts or surfaces in uniaxial crystals are a sphere and an ellipsoid and there are two points where these two wavefronts touch each other. The direction of the line joining these two points (Where the sphere and the ellipsoid touch each other) is the optic axis.

1.7 Optical Activity:-

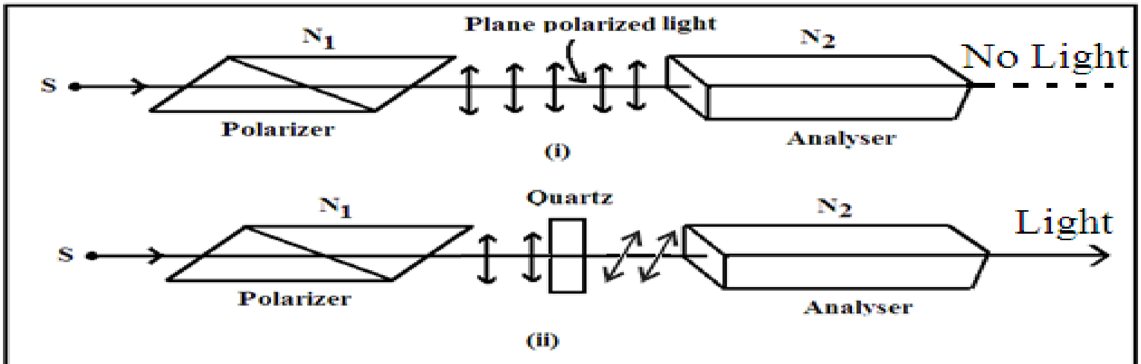


Fig. 1

When a polarizer and an analyser are crossed, no light emerges out of the analyser as shown in Fig. 1. (i). When a quartz plate cut with its faces parallel to the optic axis is introduced between N_1 and N_2 such that light falls normally upon the quartz plate, the light emerges out of N_2, as shown in Fig. 1. (ii)

The quartz plate turns the plane of vibration. The plane polarized light enters the quartz plate and its plane of vibration is gradually rotated as shown in Fig. 2.

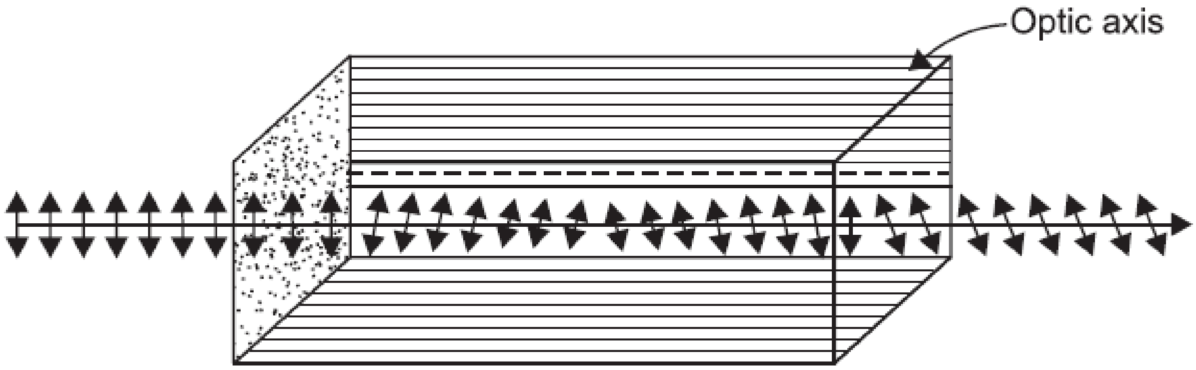


Fig. 2

The amount of rotation through which the plane of vibration is turned depends upon the thickness of the quartz plate and the wavelength of light. The action of turning the plane of vibration occurs inside the body of the plate and not on its surface. This phenomenon of rotating the plane of vibration by certain crystals or substances is known as optical activity and the substance is known as an optically active substance.

It has been found that calcite crystal does not produce any change in the plane of vibration of the plane polarised right. Therefore, it is not optically active.

Substances like sugar crystals, sugar solution, turpentine, sodium chlorate and cinnabar are optically active. Some of the substances rotate the plane of vibration to the right and they are called dextro-rotatory or right handed, the plane of vibration is rotated in a clockwise direction. The substances that rotate the plane of vibration to the left (anti-clockwise direction) are known as laevo-rotatory or left-handed.

It has been found that some quartz crystals are dextro-rotatory while others are laevo-rotatory. One is the mirror image of the other in their orientation. The rotation of the plane of vibration in a solution depends upon the concentration of the optically active substance in the solution. This helps in finding the amount of cane sugar present in a sample of sugar solution.

1.8 Fresnel's Explanation of Rotation:

A linearly polarized light can be considered as a resultant of two circularly polarized vibrations rotating in opposite directions with the same angular velocity. Fresnel assumed that a plane polarized light on entering a crystal along the optic axis is resolved into two circularly polarized vibrations rotating in opposite directions with the same angular velocity. In a crystal like calcite, the two circularly polarized vibrations travel with the same angular velocity.

Let OL is the circularly polarised vector rotating in the anti-clockwise direction and OR is the circularly polarized vector rotating in the clockwise direction as shown in Fig.(1). The resultant vector of OR and OL is OA.

According to Fresnel, when lineraly polarised light enters a crystal of calcite along the optic axis, the circularly polarized vibrations, rotating in opposite directions, have the same velocity. The resultant vibration will be along AB. Thus, crystals like calcite do not rotate the plane of vibration.

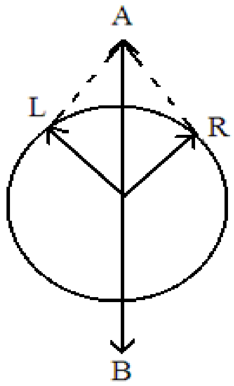


Fig. (1)

In the case of quartz crystal, the linearly polarized light, on entering the crystal is resolved into two circularly polarized vibrations rotating in opposite directions. In the case of a right-handed optically active crystal, the clockwise rotation travels faster while in a left-handed optically active crystal the anticlockwise rotation travels faster.

Considering a right-handed quartz crystal, the clockwise component travels a greater angle δ than the anticlockwise component when they emerge out of the crystal as shown in Fig (2). The resultant of these two vectors OR and OL is along OA'. Therefore, the resultant vibrations are along A'B'. Before entering the crystal, the plane of vibration is along AB and after emerging out of the crystal it is along A'B'. Therefore, the plane of vibration has rotated through an angle δ/2. The angle, through which the plane of vibration is rotated, depends upon the thickness of the crystal.

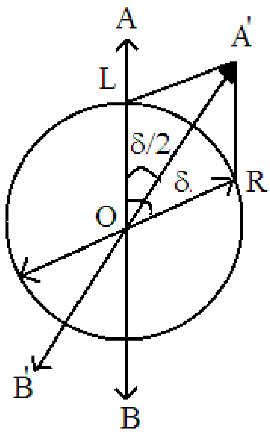


Fig. 2

Analytical treatment for Calcite crystal:

Circularly polarised light is the resultant of two rectangular components having a phase difference of π/2.

For clockwise circular vibration,

$$x_1 = a\ coswt$$

$$y_1 = a\ sinwt$$

For anti clockwise circular vibration,

$$x_2 = -a\ coswt$$

$$y_2 = a\ sinwt$$

Therefore, the resultant vibrations, along X-axis is,

$$X = x_1 + x_2 = a\ coswt - a\ coswt = 0$$

And along Y-axis,

$$Y = y_1 + y_2 = a\ sinwt + a\ sinwt = 2a\ sinwt$$

Thus, the resultant vibration has amplitude *2a* and is plane polarized. The plane of vibration is along original direction.

For Quartz crystal: In case of right-handed optically active quartz crystal, the clockwise vibration travels faster. Therefore on emerging out of the crystal, the clockwise vibrations start from R and anticlockwise vibrations start from L. The phase difference between them is δ.

For clockwise circular vibration,

$$x_1 = a\ (coswt + \delta)$$

$$y_1 = a\ (sinwt + \delta)$$

For anti clockwise circular vibration,

$$x_2 = -a\ coswt$$

$$y_2 = a\ sinwt$$

Therefore, the resultant vibrations, along X-axis is,

$$X = x_1 + x_2 = a\cos(wt + \delta) - a\ coswt = 2\text{a}\sin\left(\frac{wt + \delta - wt}{2}\right)\ \sin\left(\frac{wt + \delta + wt}{2}\right)$$

$$\therefore X = 2a\sin\left(\frac{\delta}{2}\right).\sin(wt + \frac{\delta}{2})\text{---------------------------}\ (1)$$

And along Y-axis,

$$Y = y_1 + y_2 = a\sin(wt + \delta) + a\ sinwt = 2\text{a}\cos\left(\frac{wt + \delta - wt}{2}\right)\ \sin\left(\frac{wt + \delta + wt}{2}\right)$$

$$\therefore X = 2a\cos\left(\frac{\delta}{2}\right).\sin(wt + \frac{\delta}{2})\text{---------------------------}\ (2)$$

The resultant vibrations along the X-axis and Y-axis has same phase. Therefore, the resultant vibration is plane polarised and it makes an angle δ/2 with original direction. Therefore, the plane of vibration has rotated through an angle δ/2 on passing through the crystal.

1.9 Specific Rotation:

Liquid containing an optically active substance e.g., sugar solution, camphor in alcohol etc. rotate the plane of the linearly polarized light. The angle through which the plane polarized light is rotated depends upon (1) the thickness of the medium (2) concentration of the solution or density of the active substance in the solvent. (3) Wavelength of light and (4) temperature.

The specific rotation is defined as the rotation produced by a decimeter (10 cm) long column of the liquid containing 1 gram of the active substance in one cc of the solution.

Therefore,

$$S_{\lambda}^{t} = \frac{10\theta}{lC}$$

where S_{λ}^{t} represents the specific rotation at temperature t°C for a wavelength λ, θ is the angle of rotation, l is the length of the solution in cm through which the plane polarised light passes and C is the concentration of the active substance in g/cc in the solution.

1.10 Laurent's Half Shade Polarimeter

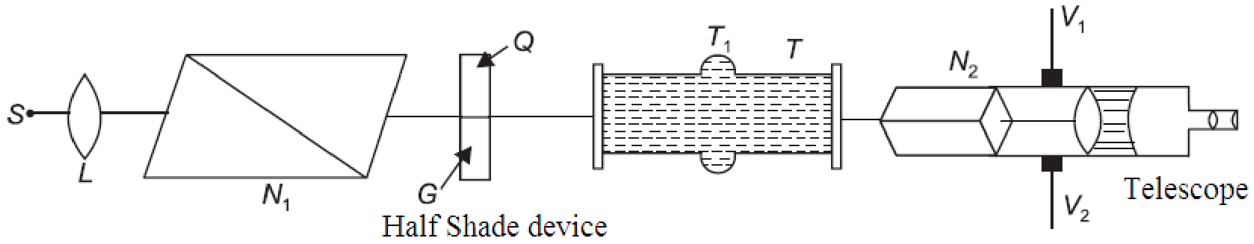


Fig. (1)

Experimental arrangement to determine specific rotation of optically active substance using Laurent's half shade polarimeter is as shown in fig. 1.

It consists of two nicol prism N_1 and N_2. N_1 is a polarizer and N_2 is an analyser. Behind N_1, there is a half wave plate of quartz Q which covers one half of the field of view while the other half G is a glass plate. The glass plate G absorbs the same amount of light as the quartz plate Q. T is a hollow glass tube having a large diameter at its middle portion. When this tube is filled with the solution containing an optically active substance and closed at the ends by cover-slips and metal covers, there will be no air bubbles in the path of light. The air bubbles (if any) will appear at the upper portion of the wide bore T_1 of the tube.

Light from a monochromatic source S is incident on the converging lens L. After passing through N_1, the beam is plane polarized. One half of the beam passes through the quartz plate Q and the other half passes through the glass plate G. Suppose the plane of vibration of the plane polarized light incident on the half shade plate is along *AB* as shown in Fig 2. Here *AB* makes an angle θ with YY′. On passing through the quartz plate *Q*, the beam is split up into ordinary and extraordinary components which travel along the same direction but with different speeds and on emergence a phase difference of π or a path difference of *λ/2* is introduced between them. The vibration of the beam emerging out of quartz will be along *CD* whereas the vibrations of the beam emerging out of the glass plate will be along *AB*. If the analyser N_2 has its principal plane or section along YY′ i.e., along the direction which bisects the angle *AOC*, the amplitudes of light incident on the analyzer N_2 from both the halves (i.e., quartz half and glass half) will be equal. Therefore, the field of view will be equally bright as shown in Fig.3 (i).

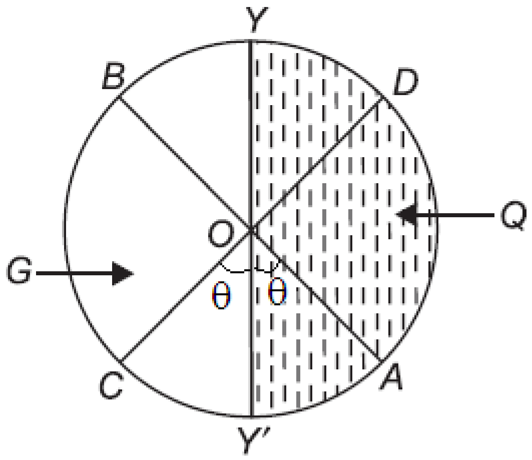


Fig. 2.

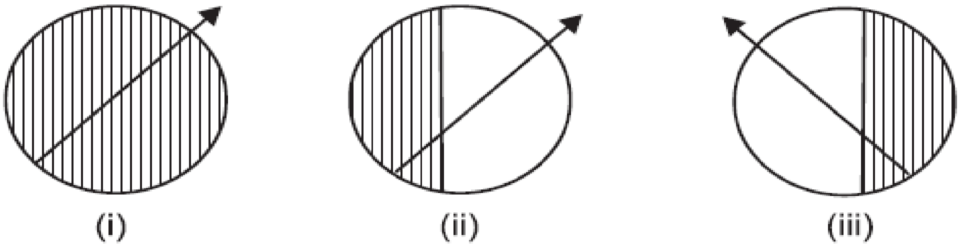


Fig. 3

If the analyser N_2 is rotated to the right of YY′ , then the right half will be brighter as compared to the left half (Fig 3.*ii*) on the other hand, if the analyser N_2 is rotated to the left of YY′ , the left half is brighter as compared to the right half (Fig. 3 *iii*).

Therefore, to find the specific rotation of an optically active substance (say sugar solution), the analyser N_2 is set in the position for equal brightness of the field of view, first without the solution in the tube *T*. The readings of the verniers V_1 and V_2 are noted. When a tube containing the solution of known concentration is placed, the vibrations from the quartz half and the glass half are rotated. In the case of sugar solution, *AB* and *CD* are rotated in the clockwise direction. Therefore, on the introduction of the tube containing the sugar solution, the field of view is not equally bright. The analyser is rotated in the clockwise direction and is brought to a position so that the whole field of view is equally bright. The new portions of the verniers V_1 and V_2 on the circular scale are read. Thus, the angle through which the analyser has been rotated gives the angle through which the plane of vibration of the incident beam has been rotated by the sugar solution. In the actual experiment, for various concentration of the sugar solution, the corresponding angles of rotation are determined. A graph is plotted between concentration *C* and the angle of rotationθ. The graph is a straight line as shown in Fig.4.

Then from the relation,

$$S_{\lambda}^{t} = \frac{10\theta}{lC}$$

the specific rotation of the optically active substance is calculated.

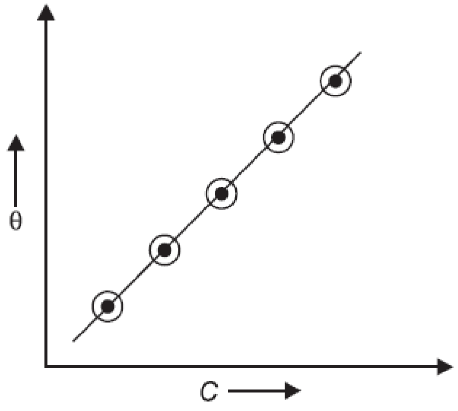


Fig.4