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Unit – I Operational Research

INTRODUCTION:

Operational Research is a method of mathematically based analysis for providing a quantities basis for management decisions. It is an analytical method of problem-solving and decision-making that is useful in the management of organizations. In operations research, problems are broken down into basic components and then solved in defined steps by mathematical analysis. Operations Research was coined during World War I. The British military brought together a group of scientists to allocate insufficient resources — for example, food, medics, weapons, troops, etc. in the most effective way possible to different military operations.

DEFINITIONS:

Operational Research defined as

1) "It is the application of scientific methods, techniques and tools to problems involving the operations of asystem so as to provide those in the control of the system with optimum solutions to the problems"
2) "Operation Research is a tool for taking decisions which searches for the optimum results in parity with theoverall objectives and constraints of the organization"
3)"It is a scientific method of providing executive department with a quantitative basis of decisionsregarding the operations under their control".
4) " It is a scientific approach to problem solving for management"
5) " It is an aid for executive in making his decisions by providing him with the needed quantitative information's based on the scientific method of analysis"

BASICS OF OPERATIONAL RESEARCH

The basics of process of operations research are as under

I – Problem Identification - To identify problem that needs to be solved.

II Model Construction - To construct model around the problem that resembles the real world and variables.

III – Use Model for Problem Solving – To use the model to derive solutions to the problem.

IV Test the Model - To test each solution on the model and analyzing its success.

V Implementation - To implement the model for solution to the actual problem.

BASICS APPLICATIONS IN DECISIONS MAKING WITH THE OPERATION RESEARCH:

The main objective of operation research is to provide better quantitative information's for making decision. Now our aim is to learn how we can have better decisions.

I- Judgment Phase:

i. Determination of operation.
ii. Determination of objectives.
iii. Determination of effectiveness of measures.
iv. Determination of type of problem, its origin and causes.

II Research Phase:

i. Observation and data collection for better understanding of the problem.
ii. Formulation of relevant hypothesis and models.
iii. Analysis of available information and verification of hypothesis.
iv. Production and generation of results and consideration of alternatives

Action Phase:

Recommendations for remedial action to those who first posed the problem, this includes the assumptions made, scope and limitations, alternative courses of action and their effect.

Putting the solution to work: implementation.

Characteristics of operations research

There are three primary characteristics of all operations research efforts:

1. Optimization- The purpose of operations research is to achieve the best performance under the given circumstances. Optimization also involves comparing and narrowing down potential options.

2. Simulation- This involves building models or replications in order to try out and test solutions before applying them.

3. Probability and statistics- This includes using mathematical algorithms and data to uncover helpful insights and risks, make reliable predictions and test possible solutions.

LINEAR PROGRAMING:

INTRODUCTION:

Linear programming is **used for obtaining the most optimal solution for a problem with given constraints**. In linear programming, we formulate our real-life problem into a mathematical model. It involves an objective function, linear inequalities with subject to constraints.

Linear programming, **mathematical modelling technique in which a linear function is maximized or minimized when subjected to various constraints**. This technique has been useful for guiding quantitative decisions in business planning, in industrial engineering, and—to a lesser extent—in the social and physical sciences. Linear programming is used as a mathematical method for determining and planning for the best outcomes and was developed during World War II by Leonid Kantorovich in 1937. It was a method used to plan expenditures and returns in a way that reduced costs for the military and possibly caused the opposite for the enemy.

Linear programming is part of an important area of mathematics called "optimization techniques" as it is literally used to find the most optimized solution to a given problem. A very basic example of linear optimization usage is in logistics or the "method of moving things around efficiently." For example, suppose there are 1000 boxes of the same size of 1 cubic meter each; 3 trucks that are able to carry 100 boxes, 70 boxes and 40 boxes respectively; several possible routes; and 48 hours to deliver all the boxes. Linear programming provides the mathematical equations to determine the optimal truck loading and route to be taken in order to meet the requirement of getting all boxes from point A to B with the least amount of going back and forth and, of course, the lowest cost at the fastest time possible.

The basic components of linear programming are as follows:

- Decision variables - These are the quantities to be determined.
- Objective function - This represents how each decision variable would affect the cost, or, simply, the value that needs to be optimized.
- Constraints - These represent how each decision variable would use limited amounts of resources.
- Data - These quantify the relationships between the objective function and the constraints.

Linear Function

"A linear function is a mathematical expression which, when graphed, will form a straight line. A linear function is a simple function usually composed of constants and simple variables without exponents as in the example, y = mx + b.

This type of function is popular in economics because of its simplicity and ease in handling. A linear function is literally a formula for a straight line when solved and all the variables are replaced with constants. The base equation of a linear function is y = mx + b where:

- "y" is the dependent variable; usually the one we are solving for so it is located to the left of the equal sign
- "x" is the independent which we manipulate to get different result of y
- "m" is the coefficient of the independent variable which determines the rate of change of "y"·
- "b" is the constant term or the y intercept

In a linear equation, if you increment the independent variable and plot the points on a graph, you get a straight line.

Solution of Linear Program

https://youtu.be/6B-ldjR-WsY

Graphical solution of linear program

https://youtu.be/qQFAvPF2OSI

Simplex Method

The Simplex method is an approach to solving linear programming models by hand using slack variables, tableaus, and pivot variables as a means to finding the optimal solution of an optimization problem. A linear program is a method of achieving the best outcome given a maximum or minimum equation with linear constraints. Most linear programs can be solved using an online solver such as MatLab, but the Simplex method is a technique for solving linear programs by hand. To solve a linear programming model using the Simplex method the following steps are necessary:

- Standard form
- Introducing slack variables
- Creating the tableau
- Pivot variables
- Creating a new tableau
- Checking for optimality
- Identify optimal values

This document breaks down the Simplex method into the above steps and follows the example linear programming model shown below throughout the entire document to find the optimal solution.

$$
\begin{aligned}
Minimize: -z &= -8x_1 - 10x_2 - 7x_3 \\
s.t.: x_1 + 3x_2 + 2x_3 &\leq 10 \\
-x_1 - 5x_2 - x_3 &\geq -8 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}
$$

Step 1: Standard Form

Standard form is the baseline format for all linear programs before solving for the optimal solution and has three requirements: (1) must be a maximization problem, (2) all linear constraints must be in a less-than-or-equal-to inequality, (3) all variables are non-negative. These requirements can always be satisfied by transforming any given linear program using basic algebra and substitution. Standard form is necessary because it creates an ideal starting point for solving the Simplex method as efficiently as possible as well as other methods of solving optimization problems.

To transform a minimization linear program model into a maximization linear program model, simply multiply both the left and the right sides of the objective function by -1.

$$-1 \times (-z = -8x_1 - 10x_2 - 7x_3)$$

$$z = 8x_1 + 10x_2 + 7x_3$$

$$Maximize: z = 8x_1 + 10x_2 + 7x_3$$

Transforming linear constraints from a greater-than-or-equal-to inequality to a less-than-or-equal-to inequality can be done similarly as what was done to the objective function. By multiplying by -1 on both sides, the inequality can be changed to less-than-or-equal-to.

$$-1 \times (-x_1 - 5x_2 - x_3 \geq -8)$$

$$x_1 + 5x_2 + x_3 \leq 8$$

Once the model is in standard form, the slack variables can be added as shown in Step 2 of the Simplex method.

Step 2: Determine Slack Variables

Slack variables are additional variables that are introduced into the linear constraints of a linear program to transform them from inequality constraints to equality constraints. If the model is in standard form, the slack variables will always have a +1 coefficient. Slack variables are needed in the constraints to transform them into solvable equalities with one definite answer.

$$x_1 + 3x_2 + 2x_3 + \mathbf{s_1} = 10$$
$$x_1 + 5x_2 + x_3 + \mathbf{s_2} = 8$$
$$x_1, x_2, x_3, \mathbf{s_1}, \mathbf{s_2} \geq 0$$

After the slack variables are introduced, the tableau can be set up to check for optimality as described in Step 3.

Step 3: Setting up the Tableau

A Simplex tableau is used to perform row operations on the linear programming model as well as to check a solution for optimality. The tableau consists of the coefficient corresponding to the linear constraint variables and the coefficients of the objective function. In the tableau below, the bolded top row of the tableau states what each column represents. The following two rows represent the linear constraint variable coefficients from the linear programming model, and the last row represents the objective function variable coefficients.

$$Maximize : z = 8x_1 + 10x_2 + 7x_3$$
$$s.t. : x_1 + 3x_2 + 2x_3 + s_1 = 10$$
$$x_1 + 5x_2 + x_3 + s_2 = 8$$

x1	**x2**	**x3**	**s1**	**s2**	**z**	**b**
1	3	2	1	0	0	10
1	5	1	0	1	0	8
-8	-10	-7	0	0	1	0

Once the tableau has been completed, the model can be checked for an optimal solution as shown in Step 4.

Step 4: Check Optimality

The optimal solution of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. The optimal solution would exist on the corner points of the graph of the entire model. To check optimality using the tableau, all values in the last row must contain values greater than or equal to zero. If a value is less than zero, it means that variable has not reached its optimal value. As seen in the previous tableau, three negative values exists in the bottom row indicating that this solution is not optimal. If a tableau is not optimal, the next step is to identify the pivot variable to base a new tableau on, as described in Step 5.

Step 5: Identify Pivot Variable

The pivot variable is used in row operations to identify which variable will become the unit value and is a key factor in the conversion of the unit value. The pivot variable can be identified by looking at the bottom row of the tableau and the indicator. Assuming that the solution is not optimal, pick the smallest negative value in the bottom row. One of the values lying in the column of this value will be the pivot variable. To find the indicator, divide the beta values of the linear constraints by their corresponding values from the column containing the possible pivot variable. The intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row will become the pivot variable.

In the example shown below, -10 is the smallest negative in the last row. This will designate the x_2 column to contain the pivot variable. Solving for the indicator gives us a value of $\frac{10}{3}$for the first constraint, and a value of $\frac{8}{5}$for the second constraint. Due to $\frac{8}{5}$ being the smallest non-negative indicator, the pivot value will be in the second row and have a value of 5.

x1	x2	x3	s1	s2	z	b	Indicator
1	3	2	1	0	0	10	10/3
1	5	1	0	1	0	8	8/5
-8	-10	-7	0	0	1	0	

Smallest Value

Now that the new pivot variable has been identified, the new tableau can be created in Step 6 to optimize the variable and find the new possible optimal solution.

Step 6: Create the New Tableau

The new tableau will be used to identify a new possible optimal solution. Now that the pivot variable has been identified in Step 5, row operations can be performed to optimize the pivot variable while keeping the rest of the tableau equivalent.

I. To optimize the pivot variable, it will need to be transformed into a unit value (value of 1). To transform the value, multiply the row containing the pivot variable by the reciprocal of the pivot value. In the example below, the pivot variable is originally 5, so multiply the entire row by $\frac{1}{5}$.

x1	x2	x3	s1	s2	z	b	
1/5	1	1/5	0	1/5	0	8/5	Pivot row

II. After the unit value has been determined, the other values in the column containing the unit value will become zero. This is because the x_2 in the second constraint is being optimized, which requires x_2 in the other equations to be zero.

x1	x2	x3	s1	s2	z	b
	0					
1/5	1	1/5	0	1/5	0	8/5
	0					

Pivot row

Pivot Column

III. In order to keep the tableau equivalent, the other variables not contained in the pivot column or pivot row must be calculated by using the new pivot values. For each new value, multiply the negative of the value in the old pivot column by the value in the new pivot row that corresponds to the value being calculated. Then add this to the old value from the old tableau to produce the new value for the new tableau. This step can be condensed into the equation on the next page:

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value)

Old Tableau:

x1	x2	x3	s1	s2	z	b
1	3	2	1	0	0	10
1	5	1	0	1	0	8
-8	-10	-7	0	0	1	0

Old pivot column

New Tableau:

x1	x2	x3	s1	s2	z	b
2/5	0	7/5	1	-3/5	0	26/5
1/5	1	1/5	0	1/5	0	8/5
-6	0	-5	0	2	1	16

New pivot row

Numerical examples are provided below to help explain this concept a little better.

Numerical examples:

I. *To find the s_2 value in row 1:*

New tableau value = (Negative value in old tableau pivot column) * (value in new tableau pivot row) + (Old tableau value)

New tableau value = (-3) * ($\frac{1}{5}$) + 0 = -$\frac{3}{5}$

II. *To find the x_1 variable in row 3:*
New tableau value = (Negative value in old tableau pivot column) * (value in new tableau pivot row) + (Old tableau value)

New value = (10) * ($\frac{1}{5}$) + -8 = -6

Once the new tableau has been completed, the model can be checked for an optimal solution.

Step 7: Check Optimality

As explained in Step 4, the optimal solution of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. Optimality will need to be checked after each new tableau to see if a new pivot variable needs to be identified. A solution is considered optimal if all values in the bottom row are greater than or equal to zero. If all values are greater than or equal to zero, the solution is considered optimal and Steps 8 through 11 can be ignored. If negative values exist, the solution is still not optimal and a new pivot point will need to be determined which is demonstrated in Step 8.

Step 8: Identify New Pivot Variable

If the solution has been identified as not optimal, a new pivot variable will need to be determined. The pivot variable was introduced in Step 5 and is used in row operations to identify which variable will become the unit value and is a key factor in the conversion of the unit value. The pivot variable can be identified by the intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row.

x1	x2	x3	s1	s2	z	b	Indicator
2/5	0	7/5	1	−3/5	0	26/5	(26/5) / (2/5) = 13
1/5	1	1	0	1/5	0	8/5	(8/5) / (1/5) = 8
-6	0	-5	0	2	1	0	

Smallest Value

With the new pivot variable identified, the new tableau can be created in Step 9.

Step 9: Create New Tableau

After the new pivot variable has been identified, a new tableau will need to be created. Introduced in Step 6, the tableau is used to optimize the pivot variable while keeping the rest of the tableau equivalent.

I. Make the pivot variable 1 by multiplying the row containing the pivot variable by the reciprocal of the pivot value. In the tableau below, the pivot value was $\frac{1}{5}$, so everything is multiplied by 5.

x1	x2	x3	s1	s2	z	b
1	5	1	0	1	0	8

II. Next, make the other values in the column of the pivot variable zero. This is done by taking the negative of the old value in the pivot column and multiplying it by the new value in the pivot row. That value is then added to the old value that is being replaced.

x1	x2	x3	s1	s2	z	b
0	-2	1	1	-1	0	2
1	5	1	0	1	0	8
0	30	1	0	8	1	64

Step 10: Check Optimality

Using the new tableau, check for optimality. Explained in Step 4, an optimal solution appears when all values in the bottom row are greater than or equal to zero. If all values are greater than or equal to zero, skip to Step 12 because optimality has been reached. If negative values still exist, repeat steps 8 and 9 until an optimal solution is obtained.

Step 11: Identify Optimal Values

Once the tableau is proven optimal the optimal values can be identified. These can be found by distinguishing the basic and non-basic variables. A basic variable can be classified to have a single 1 value in its column and the rest be all zeros. If a variable does not meet this criteria, it is considered non-basic. If a variable is non-basic it means the optimal solution of that variable is zero. If a variable is basic, the row that contains the 1 value will correspond to the beta value. The beta value will represent the optimal solution for the given variable.

x1	x2	x3	s1	s2	z	b
0	-2	1	1	-1	0	2
1	5	1	0	1	0	8
0	30	1	0	8	1	64

Basic variables: x_1, s_1, z   Non-basic variables: x_2, x_3, s_2

For the variable x_1, the 1 is found in the second row. This shows that the optimal x_1 value is found in the second row of the beta values, which is 8.

Variable s_1 has a 1 value in the first row, showing the optimal value to be 2 from the beta column. Due to s_1 being a slack variable, it is not actually included in the optimal solution since the variable is not contained in the objective function. The zeta variable has a 1 in the last row. This shows that the maximum objective value will be 64 from the beta column. The final solution shows each of the variables having values of:

x_1	= 8	s_1	= 2
x_2	= 0	s_2	= 0
x_3	= 0	z	= 64

The maximum optimal value is 64 and found at (8, 0, 0) of the objective function.

Solution of LPP using Simplex Method Videos

https://youtu.be/S1s6sWtoetg   https://youtu.be/djlnSXMIwIU